Int J Theor Phys (2009) 48: 10–13
DOI 10.1007/s10773-008-9774-2

Kaluza-Klein Cosmological Model in Self-Creation Cosmology

D.R.K. Reddy · R.L. Naidu

Received: 6 April 2008 / Accepted: 3 June 2008 / Published online: 17 June 2008

Abstract A five dimensional Kaluza-Klein Space-time is considered in the presence of a perfect fluid source in Barber's (Gen. Relativ. Gravit. 14:117, 1982) second self-creation theory of gravitation. An exact cosmological model is presented using a relation between the metric potentials and an equation of state. Some physical and kinematical properties of the model are also discussed.

Keywords Kaluza-Klein model · Self-creation cosmology

1 Introduction

In recent years, there has been a considerable interest in alternative theories of gravitation. Brans and Dicke [1] formulated a scalar-tensor theory of gravitation which incorporates Mach's principle in a relativistic frame work by assuming interaction of inertial masses of fundamental particles with some cosmic scalar field coupled with the large scale distribution matter in motion. In an attempt to produce a continuous creation theory, Barber [2] has proposed two theories. The first is a modifield Brans-Dicke theory that is unsatisfactory since the equivalence principle is violated [3]. The second is an adoption of general relativity to include continuous creation of matter and is within the limits of observation. These modifield theories create the universe out of self-contained gravitational and matter fields.

Several authors have studied various cosmological models in Baber's second self-creation theory. Pimentel [4] and Soleng [5] have discussed FRW models by using a power law relation between the expansion factor of the universe and the scalar field while Singh [6], Reddy [7, 8] and Reddy et al. [9], Reddy and Venkateswarlu [10] Shanti and Rao [11] and

D.R.K. Reddy (✉)
Department of Science and Humanities, MVGR College of Engineering, Vizianagaram 535005, India
e-mail: reddy_einstein@yahoo.com

R.L. Naidu
Department of Basic Science and Humanities, GMR Institute of Technology, Rajam 532127, India

Pradhan and Vishwakarma [12] are some of the authors who have discussed various Bianchi-type cosmological models in second self-creation theory. However, five dimensional cosmological models in the presence of perfect fluid source have not been investigated in second self-creation theory proposed by Barber. Study of higher-dimensional cosmological models are also important because of the underlying idea that the cosmos at its early stage of evolution might have had a higher dimensional era. The extra space reduced to a volume with the passage of time which is beyond the ability of experimental observation at the moment.

In this paper, we obtain a Five-dimensional Kaluza-Klein cosmological model in Barber's second self-creation theory in the presence of perfect fluid source.

2 Field Equations and the Model

We consider five dimensional Kaluza-Klein space-time given by

$$ds^2 = dt^2 - R^2\left(dx^2 + dy^2 + dz^2\right) - A^2 d\psi^2 \tag{1}$$

where $R = R(t)$ and $A = A(t)$.

The field equations in Barber's [2] second self-creation theory are

$$R_{ij} - \frac{1}{2}g_{ij}R = -8\pi\phi^{-1}T_{ij} \tag{2}$$

and

$$\Box\,\phi = \phi_{;k}^{;k} = \frac{8\pi}{3}\lambda T \tag{3}$$

where T is the trace of the energy-momentum tensor, λ is a coupling constant to be determined from the experiment $((|\lambda| \leq 0.1)$ and semi-colon denotes covariant differentiation. In the limit as $\lambda \to 0$, this theory approaches the standard general relativity theory in every respect and $G = \phi^{-1}$.

The energy-momentum tensor T_{ij} for a perfect fluid distribution is given by

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij} \tag{4}$$

where p is the isotropic pressure, ρ the energy density and u_i represents the four velocity of the fluid. Corresponding to the metric given by (1), the four velocity vector u_i satisfies the equation

$$g_{ij}u^i u^j = 1 \tag{5}$$

In a comoving coordinate system the field equations (2) and (3) for the metric (1) with the help of (4) and (5) take the form

$$3\left(\frac{R_4}{R}\right)^2 + 3\frac{R_4 A_4}{RA} = 8\pi\phi^{-1}\rho \tag{6}$$

$$2\frac{R_{44}}{R} + \frac{A_{44}}{A} + 2\frac{R_4 A_4}{RA} + \frac{R_4^2}{R^2} = -8\pi\phi^{-1}p \tag{7}$$

$$3\frac{R_{44}}{R} + 3\left(\frac{R_4}{R}\right)^2 = -8\pi\phi^{-1}p \tag{8}$$

$$\phi_{44} + \phi_4\left(3\frac{R_4}{R} + \frac{A_4}{A}\right) = \frac{8\pi}{3}\lambda(\rho - 4p) \tag{9}$$

where the suffix 4 following an unknown function denotes ordinary differentiation with respect to time t.

The field equations (6)–(9) are four independent equations in five unknowns R, A, ρ, p and ϕ. Hence to get a determinate solution one has to assume a physical or mathematical condition. We solve the above set of field equations with the equation of state

$$\rho = 4p \tag{10}$$

which is analogous to the equation of state $\rho = 3p$ which represents disordered radiation in four dimensional space. To obtain a determinate solution we also use a relation between the metric potentials

$$A = \mu R^n \tag{11}$$

where μ and n are constants.

Now, using (10) and (11) the field equations (6)–(9) yield an exact solution given by

$$\begin{aligned} &R = [(n+3)(at+b)]^{1/n+3} \\ &A = \mu[(n+3)(at+b)]^{n/n+3}, \quad n+3 \neq 0 \end{aligned} \tag{12}$$

$$8\pi\phi^{-1}\rho = 8\pi\phi^{-1}(4p) = 3a^2\left[\frac{n+1}{(n+3)^2}\right]\frac{1}{(at+b)^2} \tag{13}$$

and

$$\phi = \frac{k}{a\,(n+3)}\log(at+b) + \phi_0 \tag{14}$$

where a, b, k and ϕ_0 are constants of integration and n satisfies the relation

$$4n^2 + 19n + 19 = 0 \tag{15}$$

Thus, five dimensional Kaluza-Klein cosmological model corresponding to the above solution can be written (after a suitable choice of coordinates and constants of integration) as

$$ds^2 = dT^2 - [(n+3)T]^{2/n+3}(dX^2 + dY^2 + dZ^2) - \mu^2[(n+3)T]^{2n/n+3}d\psi^2 \tag{16}$$

3 Some Physical Properties of the Model

Equation (16) represents an exact five dimensional Kaluza-Klein cosmological model in the frame work of second self-creation theory of gravitation, proposed by Barber [2], in the presence of a perfect fluid source. We observe that the model has no initial singularity for $n + 3 \neq 0$.

For the model (16), the physical and kinematical variables which are important, in cosmology, are

$$8\pi\rho = 8\pi\,(4p) = \frac{3(n+1)}{(n+3)^2T^2}\left[\frac{1}{n+3}\log T + \phi_0\right] \tag{17}$$

$$\phi = \left(\frac{1}{n+3}\right)\log T + \phi_0 \tag{18}$$

$$\text{Spatial volume:} \quad V^3 = \mu(n+3)T \tag{19}$$

$$\text{Expansion scalar:} \quad \theta = \frac{1}{3T} \tag{20}$$

$$\text{Shear scalar:} \quad \sigma^2 = \frac{1}{54T^2} \tag{21}$$

$$\text{Deceleration parameter:} \quad q = 8 \tag{22}$$

$$\text{Hubble's parameter:} \quad H = \frac{1}{(n+3)T} \tag{23}$$

Equation (19) shows the anisotropic expansion of the universe (16) with time for $n+3>0$. The energy density ρ, the isotropic pressure p tend to zero as time increases indefinitely. For this model the expansion scalar θ, shear scalar σ and Hubble's parameter H tend to zero as $T \to \infty$. The positive value of the deceleration parameter q shows that the model decelerates in the standard way. However, the model does not admit rotation and acceleration. Hence the model (16) represents expanding, shearing, non-rotating and non-singular universe which decelerates in the standard way. However the scalar field diverges as T increases indefinitely. Also, since

$$\operatorname*{Lim}_{T\to\infty} \frac{\sigma^2}{\theta^2} \neq 0$$

the model does not approach isotropy for large T.

4 Conclusions

Here we have presented a five dimensional Kaluza-Klein cosmological model in Barber's second-self creation theory of gravitation in the presence of perfect fluid source. The model obtained is free from initial singularity. The model is expanding, shearing, non-rotating and decelerates in the standard way.

References

1. Brans, C., Dicke, R.H.: Phys. Rev. **124**, 925 (1961). doi:10.1103/PhysRev.124.925
2. Barber, G.A.: Gen. Relativ. Gravit. **14**, 117 (1982). doi:10.1007/BF00756918
3. Brans, C.: Gen. Relativ. Gravit. **19**, 949 (1987). doi:10.1007/BF00759299
4. Pimentel, L.O.: Astrophys. Space Sci. **116**, 935 (1985)
5. Soleng, H.H.: Astrophys. Space Sci. **139**, 13 (1987). doi:10.1007/BF00643809
6. Tarkeshwar, S.: Astrophys. Space Sci. **102**, 67 (1984). doi:10.1007/BF00651062
7. Reddy, D.R.K.: Astrophys. Space Sci. **133**, 189 (1987). doi:10.1007/BF00637431
8. Reddy, D.R.K.: Astrophys. Space Sci. **133**, 389 (1987). doi:10.1007/BF00642496
9. Reddy, D.R.K., Avadhanulu, M.B., Venkateswarlu, R.: Astrophys. Space Sci. **134**, 201 (1987). doi:10.1007/BF00636469
10. Reddy, D.R.K., Venkateswarlu, R.: Astrophys. Space Sci. **155**, 135 (1989). doi:10.1007/BF00645214
11. Shanti, K., Rao, V.U.M.: Astrophys. Space Sci. **179**, 147 (1991). doi:10.1007/BF00642359
12. Pradhan, A., Vishwakarma, A.K.: Int. J. Mod. Phys. D **11**, 1195 (2002). doi:10.1142/S0218271802002207